A conditional statement is a statement that can be written as an if-then statement, “if $p$, then $q$.”

The **hypothesis** comes after the word *if.*

The **conclusion** comes after the word *then.*

If you buy this cell phone, then you will receive 10 free ringtone downloads.

Sometimes it is necessary to rewrite a conditional statement so that it is in if-then form.

**Conditional:** A person who practices putting will improve her golf game.

**If-Then Form:** If a person practices putting, then she will improve her golf game.

A conditional statement has a false **truth value** only if the hypothesis (H) is true and the conclusion (C) is false.

For each conditional, underline the hypothesis and double-underline the conclusion.

1. If $x$ is an even number, then $x$ is divisible by 2.
2. The circumference of a circle is $5\pi$ inches if the diameter of the circle is 5 inches.
3. If a line containing the points $J, K, \text{ and } L$ lies in plane $P$, then $J, K, \text{ and } L$ are coplanar.

For Exercises 4–6, write a conditional statement from each given statement.

4. Congruent segments have equal measures.

5. On Tuesday, play practice is at 6:00.

6. **Adjacent Angles**
   **Linear Pair**

Determine whether the following conditional is true. If false, give a counterexample.

7. If two angles are supplementary, then they form a linear pair.
The negation of a statement, “not $p$,” has the opposite truth value of the original statement.

If $p$ is true, then $\neg p$ is false.
If $p$ is false, then $\neg p$ is true.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Example</th>
<th>Truth Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional</strong></td>
<td>If a figure is a square, then it has four right angles.</td>
<td>True</td>
</tr>
<tr>
<td><strong>Converse:</strong> Switch H and C.</td>
<td>If a figure has four right angles, then it is a square.</td>
<td>False</td>
</tr>
<tr>
<td><strong>Inverse:</strong> Negate H and C.</td>
<td>If a figure is not a square, then it does not have four right angles.</td>
<td>False</td>
</tr>
<tr>
<td><strong>Contrapositive:</strong> Switch and negate H and C.</td>
<td>If a figure does not have four right angles, then it is not a square.</td>
<td>True</td>
</tr>
</tbody>
</table>

Write the converse, inverse, and contrapositive of each conditional statement. Find the truth value of each.

8. If an animal is an armadillo, then it is nocturnal.

- **Converse:** If an animal is nocturnal, then it is an armadillo; false.
- **Inverse:** If an animal is not an armadillo, then it is not nocturnal; false.
- **Contrapositive:** If an animal is not nocturnal, then it is not an armadillo; true.

9. If $y = 1$, then $y^2 = 1$.

- **Converse:** If $y^2 = 1$, then $y = 1$; false.
- **Inverse:** If $y$ is not 1, then $y^2$ is not 1; false.
- **Contrapositive:** If $y^2$ is not 1, then $y$ is not 1; true.

10. If an angle has a measure less than 90°, then it is acute.

- **Converse:** If an angle is acute, then it has a measure less than 90°; true.
- **Inverse:** If an angle does not have a measure less than 90°, then it is not acute; true.
- **Contrapositive:** If an angle is not acute, then it does not have a measure less than 90°; true.
### Practice B

#### Conditional Statements

Write a conditional statement from each of the following.

1. If three points are noncollinear, then they determine a plane.

2. If a food is a kumquat, then it is a fruit.

Determine if each conditional is true. If false, give a counterexample.

3. If a liquid is water, then it is composed of hydrogen and oxygen.

4. If you can see the stars, then it is night.

5. If a whole number has three or more factors, then it is composite.

6. If a living thing is green, then it is a plant.

7. If a person practices putting, then she will improve her golf game.

8. If it rains, then I go indoors.

9. If an animal is a rodent, then it is not an ape.

10. If an animal is an ape, then it is a mammal.

### Retake

#### Conditional Statements

A conditional statement is a statement that can be written as an if-then statement, "if $p$, then $q$.”

If you buy this cell phone, then you will receive 10 free ringtones downloads.

Sometimes it is necessary to rewrite a conditional statement so that it is in if-then form.

**Conditional:** A person who practices putting will improve her golf game.

**If-Then Form:** If a person practices putting, then she will improve her golf game.

A conditional statement has a false truth value only if the hypothesis ($H$) is true and the conclusion ($C$) is false.

For each conditional, underline the hypothesis and double-underline the conclusion.

1. If $x$ is an even number, then $x$ is divisible by 2.

2. The circumference of a circle is 5π inches if the diameter of the circle is 5 inches.

3. If a line containing the points $J, K$, and $L$ is in plane $F$, then $J, K$, and $L$ are coplanar.

For Exercises 4–6, write a conditional statement from each given statement.

4. Congruent segments have equal measures.

   If segments are congruent, then they have equal measures.

5. On Tuesday, play practice is at 6:00.

   If it is Tuesday, then play practice is at 6:00.

6. If two angles form a linear pair, then they are adjacent angles.

Determine whether the following conditional is true. If false, give a counterexample.

7. If two angles are supplementary, then they form a linear pair.

   True; two supplementary angles need not be adjacent.
Challenge
22 Paradoxes

A paradox is a statement or set of statements that appears to be self-contradictory or that simply cannot be explained. For centuries, people have found paradoxes intriguing because they often arise from valid logical arguments.

For example, the statement “All Cretans are liars” is attributed to Epimenides, a Greek poet who lived in the sixth century B.C. Epimenides was a Cretan—a resident of Crete—and this gives rise to the paradox: If his statement is true, then Epimenides is a liar. Assuming that liars never tell the truth, this means that his statement is false!

Explain why each statement or set of statements creates a paradox.
1. This statement is false.
Explanations will vary. Assume that the given statement is true. Then “This statement is false” is false.

2. There are three false statements here. Identify them.
Explanations will vary. Identify them.

3. A sign posted on a wall gives this order: Do not read this sign.
Explanations will vary. Assume that the only way to know the order is to read the sign. If a person reads the sign, however, then he or she has already violated the order to not read the sign.

4. In general, the paradoxes in Exercises 1–4 arise because the key statement refers to itself. These are called self-referential paradoxes. On a separate sheet of paper, write four original statements or sets of statements that create self-referential paradoxes. Answers will vary.

5. Sometimes a picture depicts a physical paradox. Explain why the Penrose staircase pictured right is a paradox.
Explanations will vary. No matter where you start on the staircase, it appears that a walk around it takes you continually upward (or downward). However, you eventually return to the place where you started.

6. Using your library or the Internet as a resource, find two examples of paradoxes that are of a type different from those described above. Write your examples and explanations on a separate sheet of paper. Findings will vary.

Problem Solving
21 Conditional Statements

1. Write the converse, inverse, and contrapositive of the conditional statement. Find the truth value of each.

a) If it is after 1818, then the U.S. flag has less than 50 stars; false.

b) If the year is 1777, then the U.S. flag has 30 stars; false.

c) If an animal is an armadillo, then it is nocturnal. Conv.: If an animal is nocturnal, then it is an armadillo; false. Inv.: If an animal is not an armadillo, then it is not nocturnal; false. Contra.: If an animal is not nocturnal, then it is not an armadillo; true.

2. Write a conditional statement from the diagram. Then write the converse, inverse, and contrapositive. Find the truth value of each.

3. An angle has a measure less than 90°; true.
Inv.: If an angle does not have a measure less than 90°, then it is not acute; false. Contra.: If an angle is not acute, then it does not have a measure less than 90°; true.

4. If a figure is a square, then it has four right angles. Conv.: If a figure is a square, then it has four right angles; true. Inv.: If it is not a square, then it does not have four right angles; false. Contra.: If a figure does not have four right angles, then it is not a square; true.

5. If you do not have a cell phone, then you did not receive a text message. Conv.: If you do not have a cell phone, then you did not receive a text message; false. Inv.: If you did receive a text message, then you do have a cell phone; false. Contra.: If you received a text message, then you do not have a cell phone. This statement is false.

6. If you do not know how the movie ends, then you did not see the movie.
Inv.: If you did see the movie, then you do know how the movie ends; false. Contra.: If you saw the movie, then you do know how it ends; true.

Determine the converse, inverse, and contrapositive of the conditional statements. Indicate whether each statement is true or false.

1. If \( m \angle A = 140° \), then \( \angle A \) is obtuse.
Conv.: If \( \angle A \) is obtuse, then \( m \angle A = 140° \); true.

2. If \( m \angle A = 140° \), then \( \angle A \) is obtuse.
Inv.: If \( \angle A \) is obtuse, then \( m \angle A = 140° \); false.

Reading Strategies

Related Conditional Statements

<table>
<thead>
<tr>
<th>Conditional</th>
<th>Converse</th>
<th>Inverse</th>
<th>Contrapositive</th>
</tr>
</thead>
<tbody>
<tr>
<td>A conditional consists of a hypothesis ( h ) and a conclusion ( c ).</td>
<td>The converse statement is formed by exchanging the hypothesis with the conclusion.</td>
<td>The inverse statement is formed by negating the hypothesis and the conclusion.</td>
<td>The contrapositive is formed by both exchanging and negating the hypothesis and the conclusion.</td>
</tr>
</tbody>
</table>

EXAMPLE
- **If \( m \angle Z = 25° \), then \( \angle Z \) is acute.**
- \( \angle Z \) is acute. This statement is TRUE.
- \( m \angle Z = 25° \), then \( \angle Z \) is acute. This statement is FALSE.
- \( m \angle Z = 25° \), then \( \angle Z \) is not acute. This statement is TRUE.
- \( m \angle Z = 25° \), then \( \angle Z \) is not acute. This statement is FALSE.

Determine the converse, inverse, and contrapositive of the conditional statements. Indicate whether each statement is true or false.

1. \( \angle A \) is acute.
Conditional: \( m \angle A = 140° \), then \( \angle A \) is obtuse.
Converse: \( \angle A \) is obtuse, then \( m \angle A = 140° \); true.

2. \( \angle A \) is obtuse.
Conditional: \( \angle A \) is obtuse, then \( m \angle A = 140° \); false.
Converse: \( m \angle A = 140° \), then \( \angle A \) is obtuse; false.

Choose the best answer.
6. What is the converse of “If you saw the movie, then you know how it ends”? (A) If you know how the movie ends, then you saw the movie.
   (B) If you did not see the movie, then you do not know how it ends.
   (C) If you do not know how the movie ends, then you did not see the movie.
   (D) If you do not know how the movie ends, then you saw the movie.
   Choose the best answer.

7. What is the inverse of “If you received a text message, then you have a cell phone”? (A) If you have a cell phone, then you received a text message.
   (B) If you did not receive a text message, then you do not have a cell phone.
   (C) If you did not receive a text message, then you do not have a cell phone.
   (D) If you received a text message, then you do not have a cell phone.
   Choose the best answer.