LESSON 2-3
Reteach

**Using Deductive Reasoning to Verify Conjectures**

With inductive reasoning, you use examples to make a conjecture. With **deductive reasoning**, you use facts, definitions, and properties to draw conclusions and prove that conjectures are true.

**Given:** If two points lie in a plane, then the line containing those points also lies in the plane. \(A\) and \(B\) lie in plane \(N\).

**Conjecture:** \(\overline{AB}\) lies in plane \(N\).

One valid form of deductive reasoning that lets you draw conclusions from true facts is called the **Law of Detachment**.

<table>
<thead>
<tr>
<th>Given</th>
<th>If you have $2, then you can buy a snack. You have $2.</th>
<th>If you have $2, then you can buy a snack. You can buy a snack.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conjecture</td>
<td>You can buy a snack. You have $2.</td>
<td>Valid Conjecture? Yes; the conditional is true and the hypothesis is true. No; the hypothesis may or may not be true. For example, if you borrowed money, you could also buy a snack.</td>
</tr>
</tbody>
</table>

Tell whether each conclusion uses inductive or deductive reasoning.

1. A sign in the cafeteria says that a car wash is being held on the last Saturday of May. Tomorrow is the last Saturday of May, so Justin concludes that the car wash is tomorrow. __________________

2. So far, at the beginning of every Latin class, the teacher has had students review vocabulary. Latin class is about to start, and Jamilla assumes that they will first review vocabulary. __________________

3. Opposite rays are two rays that have a common endpoint and form a line. \(\overrightarrow{XY}\) and \(\overrightarrow{YZ}\) are opposite rays. __________________

Determine whether each conjecture is valid by the Law of Detachment.

4. Given: If you ride the Titan roller coaster in Arlington, Texas, then you will drop 255 feet.
   Michael rode the Titan roller coaster.
   Conjecture: Michael dropped 255 feet. __________________

5. Given: A segment that is a diameter of a circle has endpoints on the circle.
   \(\overline{GH}\) has endpoints on a circle.
   Conjecture: \(\overline{GH}\) is a diameter. __________________
LESSON Reteach

2-3 Using Deductive Reasoning to Verify Conjectures continued

Another valid form of deductive reasoning is the Law of Syllogism. It is similar to the Transitive Property of Equality.

<table>
<thead>
<tr>
<th>Transitive Property of Equality</th>
<th>Law of Syllogism</th>
</tr>
</thead>
<tbody>
<tr>
<td>If ( y = 10x ) and ( 10x = 20 ), then ( y = 20 ).</td>
<td><strong>Given:</strong> If you have a horse, then you have to feed it. If you have to feed a horse, then you have to get up early every morning. <strong>Conjecture:</strong> If you have a horse, then you have to get up early every morning.</td>
</tr>
</tbody>
</table>

Determine whether each conjecture is valid by the Law of Syllogism.

6. Given: If you buy a car, then you can drive to school. If you can drive to school, then you will not ride the bus.
   Conjecture: If you buy a car, then you will not ride the bus.
   ________________

7. Given: If \( \angle K \) is obtuse, then it does not have a measure of 90°. If an angle does not have a measure of 90°, then it is not a right angle.
   Conjecture: If \( \angle K \) is obtuse, then it is not a right angle.
   ________________

8. Given: If two segments are congruent, then they have the same measure. If two segments each have a measure of 6.5 centimeters, then they are congruent.
   Conjecture: If two segments are congruent, then they each have a measure of 6.5 centimeters.
   ________________

Draw a conclusion from the given information.

9. If \( \triangle LMN \) is translated in the coordinate plane, then it has the same size and shape as its preimage. If an image and preimage have the same size and shape, then the figures have equal perimeters. \( \triangle LMN \) is translated in the coordinate plane.

10. If \( \angle R \) and \( \angle S \) are complementary to the same angle, then the two angles are congruent. If two angles are congruent, then they are supplementary to the same angle. \( \angle R \) and \( \angle S \) are complementary to the same angle.
Practice A

Using Deductive Reasoning to Verify Conjectures

1. Inductive reasoning is using observations to find a pattern or rule. **Deductive reasoning** is making logical conclusions from statements that are known or assumed.

Tell whether each conclusion is a result of inductive or deductive reasoning.

2. The Law of Detachment states that if \( p \rightarrow q \) is a true statement and \( p \) is true, then \( q \) is true.

3. Marcus learns in Social Studies that a presidential election happens every four years. He knows that the last presidential election was in 2004, so he concludes that the next presidential election will be in 2008.

4. Cecile and her father talk about the election of President Carter in 1976 and the election of President Reagan in 1980 and 1984. She concludes that a presidential election happens every four years.

5. The Law of Syllogism states that if \( p \rightarrow q \) and \( q \rightarrow r \) are true statements, then \( p \rightarrow r \) is a true statement.

For Exercise 8, use these statements:

- \( p \): You are a clown.
- \( q \): You wear big shoes.
- \( r \): Your feet hurt.

6. Use the Law of Syllogism to complete a valid conjecture.

Given: If you are a clown, then you wear big shoes.

Conjecture: If you wear big shoes, then your feet hurt.

7. The Law of **Syllogism** states that if \( p \rightarrow q \) and \( q \rightarrow r \) are true statements, then \( p \rightarrow r \) is a true statement.

State the hypothesis (\( p \)) and conclusion (\( q \)) of each given conditional. Then tell whether the conjecture is valid or not valid by the Law of Detachment.

5. Given: If a person sees penguins, then the person is in Antarctica. Carla sees penguins.

Conjecture: Carla is in Antarctica.

Hypothesis: A person sees penguins.

Conclusion: The person is in Antarctica.

The conjecture is **valid**.

6. Given: If a person sees a polar bear, then the person is in the Arctic. James is in the Arctic.

Conjecture: James sees a polar bear.

Hypothesis: A person sees a polar bear.

Conclusion: The person is in the Arctic.

The conjecture is **invalid**.

7. The Law of **Syllogism** states that if \( p \rightarrow q \) and \( q \rightarrow r \) are true statements, then \( p \rightarrow r \) is a true statement.

For Exercise 8, use these statements:

- \( p \): You are a clown.
- \( q \): You wear big shoes.
- \( r \): Your feet hurt.

8. Use the Law of Syllogism to complete a valid conjecture.

Given: If you are a clown, then you wear big shoes. If you wear big shoes, then your feet hurt.

Conjecture: If you are a clown, then your feet hurt.

Practice B

Using Deductive Reasoning to Verify Conjectures

Tell whether each conclusion is a result of inductive or deductive reasoning.

1. The United States Census Bureau collects data on the earnings of American citizens. Using data for the three years from 2001 to 2003, the bureau concluded that the national average median income for a four-person family was $43,527.

2. A speeding ticket costs $40 plus $5 per mile over the speed limit. Frank concludes that Lynne was driving 7 mi/h over the speed limit.

Determine if each conjecture is valid by the Law of Detachment.

3. Given: If \( m_{\angle}ABC \equiv m_{\angle}CBD \), then \( \triangle BCD \sim \triangle ABD \). \( \triangle ABC \sim \triangle CBD \).

Conjecture: \( m_{\angle}ABC \equiv m_{\angle}CBD \).

4. Given: You will catch a catfish if you use stink bait. Stuart caught a catfish.

Conjecture: Stuart used stink bait.

5. Given: An obtuse triangle has two acute angles. \( \triangle ABC \) is obtuse.

Conjecture: \( \triangle ABC \) has two acute angles.

Determine if each conjecture is valid by the Law of Syllogism.

6. Given: If the gossip said it, then it must be true. If it is true, then somebody is in big trouble.

Conjecture: Somebody is in big trouble because the gossip said it.

7. Given: No human is immortal. Fido the dog is not human.

Conjecture: Fido the dog is immortal.

8. Given: The radio is distracting when I am studying. If it is 7:30 P.M. on a weekend, I am studying.

Conjecture: If it is 7:30 P.M. on a weekend, the radio is distracting.

Determine whether each conjecture is valid by the Law of Detachment.

9. Given: If two segments intersect, then they are not parallel. If two segments are not parallel, then they could be perpendicular. \( EF \) and \( MN \) intersect.

Conjecture: \( EF \) and \( MN \) could be perpendicular.

10. Given: When you are relaxed, your blood pressure is relatively low. If you are sailing, you are relaxed. Becky is sailing.

Conjecture: Becky’s blood pressure is relatively low.

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Using Deductive Reasoning to Verify Conjectures

Draw a Venn diagram to represent each situation.

1. the Law of Syllogism

2. \( p \rightarrow r \) and \( q \rightarrow r \) are true, but \( p \rightarrow q \) is not true.

3. \( p \rightarrow r \) and \( p \rightarrow q \) are true, but \( q \rightarrow r \) and \( r \rightarrow q \) are not always true.

4. \( p \rightarrow r \) and \( p \rightarrow q \) and \( r \rightarrow q \) are all true.

5. Explain how, given that \( p \rightarrow q \) and \( q \rightarrow r \) are true statements, the Law of Syllogism can be derived from the Law of Detachment.

Possible answer: The Law of Detachment states that if \( p \rightarrow q \) is true and \( p \rightarrow r \) is true, then \( q \rightarrow r \) is true. Also by the Law of Detachment, if \( q \rightarrow r \) is true and \( q \rightarrow r \) is true, then \( r \rightarrow r \) is true. So when \( p \rightarrow r \) is true, \( r \rightarrow r \) is true, and thus \( p \rightarrow r \) is true.

6. Imagine there is a “Law of Trilogism” that relates three conditional statements in a manner similar to the Law of Syllogism. Write the “Law of Trilogism.”

If \( p \rightarrow q \) and \( q \rightarrow r \) and \( r \rightarrow s \) are true statements, then \( p \rightarrow s \) is a true statement.

7. Make a conclusion from the following statements, given that the director knows how to make a movie. (Hint: The contrapositive of a conditional statement is logically equivalent to the original statement.)

   I. If a movie makes money, then a sequel will be made.
   II. Any director who knows how to make a movie films at 24 frames per second.
   III. A movie makes money when people enjoy the movie.
   IV. The director knows how to make a movie.
   V. If the human eye does not see the action as continuous, then the movie was not filmed at 24 frames per second.
   VI. People do not enjoy a movie when the human eye does not see the action as continuous.

A sequel will be made.
### Challenge

**Analyze Clues with Deductive Reasoning**

Jake, Anna, and Ling are each originally from different states: California, Texas, and New Jersey. One of the students is on Student Council, one is on the swim team, and one is in the math club. The following statements are true.

- a. Ling is not involved with student government this year, and she has never been to Texas.
- b. Anna rides the bus with the person from New Jersey.
- c. Jake is not from California.
- d. Jake likes to go to his friend’s swim meets.
- e. The person from California is Ling’s neighbor. It is the person she goes to for math questions.
- f. The person on Student Council was born in the same month as Anna.

#### Use the Information to Answer Each Question.

1. **Use statement b to formulate a conditional statement that is helpful in solving this puzzle.**
   - **Possible answer:** If Anna rides the bus with the person from New Jersey, then Anna is not from New Jersey. So Anna must be from California or Texas.

2. **Determine whether or not each conjecture is valid. Explain.**
   - a. If a person is not involved with student government, then he or she is not on Student Council. Ling is not involved with student government. So Ling is not on Student Council.
   - **Yes:** the conditional is true, and the hypothesis is true.
   - b. If a person is not from the West Coast, then he or she is not from California. Jake is not from California. Therefore, Jake is not from the West Coast.
   - **No:** a true conditional and a true conclusion do not necessarily make the hypothesis true. Jake could be from a different West Coast state.

3. **Use one of the statements listed above to write a conditional statement and make a valid conjecture. Describe how it is helpful in solving the puzzle.**
   - **Possible answer:** If Jake likes to go to his friend’s swim meets, then he is not on the swim team. **Conjecture:** Jake is not on the swim team. So Jake is on Student Council or in the math club.

4. **Apply what you know about deductive reasoning to tell where Jake, Anna, and Ling are from and in which activity they are involved.**
   - Jake is from Texas and is on Student Council. Anna is from California and is in the math club. Ling is from New Jersey and is on the swim team.

### Problem Solving

**Using Deductive Reasoning to Verify Conjectures**

Use the information in the table and the given statement to draw a valid conclusion for each. A valid conclusion cannot be made, explain why not.

#### Graphic Organizer

**Reading Strategies**

Conjectures can be verified by using deductive reasoning.

#### Graphic Organizer

**Using Deductive Reasoning to Verify Conjectures**

**Volcanic Eruptions**

| I. | A category 2 eruption produces a plume of ash 1–5 kilometers high. |
| II. | An explosive volcano produces a volume of ash between 1 million and 10 million cubic meters. |
| III. | A volume of ash 10,000–1,000,000 cubic meters is produced, the eruption is classified as a category 1 eruption. |
| IV. | If the eruption is severe, it produces a plume of ash between 3 and 6 kilometers high. |

1. **Given:** When Mt. Kilauea in Hawaii erupted, it produced a volume of ash between 10,000 and 1 million cubic meters.

   **Conclusion:** Mt. Kilauea’s eruption in Hawaii was a category 1 eruption.

2. **Given:** The eruption of a volcano in Unzen, Japan, was not explosive.

   **No valid conclusion can be made. Possible answer:** The negation of the hypothesis does not produce a valid conclusion given a true conditional.

3. **Given:** The eruption of a volcano in Stromboli, Italy, was a category 2 eruption.

   **Conclusion:** The eruption of the volcano in Stromboli, Italy, produced a plume of ash 1–5 kilometers high.

4. **A sports store has running shoes 25% off original prices. Andrea sees a pair of running shoes that she likes for $65.00. Which is a valid conclusion?**
   - **A.** The sale price of the shoes is $48.75.
   - **B.** Andrea will buy the shoes.
   - **C.** Andrea will not buy the shoes.
   - **D.** Andrea will buy the shoes. (Valid conclusion)

5. If Zack makes $\frac{1}{2}$ quarts of lemonade, then he uses 6 lemons. If Zack makes 1$\frac{1}{2}$ quarts of lemonade, then he makes 4 servings. Zack uses 5 lemons. Which is a valid conclusion?
   - **F.** Zack makes 3 servings.
   - **G.** Zack makes 2 servings.
   - **H.** Zack makes 1 quart.
   - **I.** Zack does not make $\frac{1}{2}$ quarts. (Valid conclusion)